Squeezing Parameter Impact on Fidelity of Wireless Microwave Quantum Teleportation

Abdurrahman Wachid Shaffar, Kyung Min Byun, and Hyundong Shin
Department of Electronics and Information Convergence Engineering
Kyung Hee University, Yongin 17104, Republic of Korea
Email: hshin@khu.ac.kr

***Abstract*—The study evaluated the impact of varying the squeezing parameter on the fidelity of quantum teleportation using microwave two-mode squeezed thermal (TMST) states as the entanglement resource. The effect of different squeezing parameters on the fidelity of teleportation protocol has been assessed through numerical simulations. The findings revealed that higher squeezing parameters enhance fidelity at short distances but result in more rapid degradation over longer distances due to thermal noise and attenuation in the open-air microwave channel. Therefore, it is crucial to determine an optimal squeezing parameter that balances high initial fidelity with minimal long-range degradation. This optimal value must be experimentally attainable to ensure the practical implementation of an efficient quantum teleportation protocol using microwave TMST states. The insights gained from this study are valuable for optimizing quantum communication systems in real-world conditions.**

I. Introduction

Superconducting qubits have emerged as one of the leading platforms in realizing quantum computers due to their unique capability for modular and distributed scaling architectures [1]. That scalability increases further optimism about the path ahead for quantum computing. Nevertheless, the vast difference in operational frequencies between superconducting qubits operating within the microwave spectrum and the typical frequencies used in current quantum networks is one of the main obstacles to progress in superconducting quantum computing, as those frequencies usually lie in the optical regime [2]. Microwave quantum communication opens an alternative route to interconnecting superconducting quantum processors and hence allows for avoiding cumbersome frequency-conversion mechanisms [3]. One of the most promising ways is wireless quantum interconnection, where several quantum computers are connected via quantum WiFi, considering the omnipresence of microwave frequencies in classic wireless communications.

Quantum teleportation is essential for distributed quantum computing and should be efficiently conducted within the microwave domain to ensure seamless communication between superconducting quantum systems [2]. This protocol relies on two-mode squeezed states generated by squeezing and correlating two coherent modes to produce an entangled state [4]. In actual applications, however, thermal noise in the hardware will turn those ideal two-mode squeezed states into two-mode squeezed thermal (TMST) states, impairing the overall performance. One critical factor affecting the generation of TMST is the squeezing parameter, which directly affects the fidelity of quantum teleportation. The optimization of this factor becomes urgent since it seriously affects the performance of teleportation. In this paper, we explore the dependence of the fidelity in microwave quantum teleportation on the squeezing parameter using TMST as the resource state. Numerically studying the effect of changing the squeezing parameter in the process provides deep insight into teleportation behavior and will be very helpful for implementing quantum communication tasks in superconducting quantum computing networks.

II. Theoretical Framework

A. Continuous-variable Quantum Teleportation Protocol

Continuous-variable (CV) quantum teleportation involves transmitting the quantum state of a mode from one party (Alice) to another (Bob) using a shared entangled resource, such as a two-mode squeezed vacuum (TMSV) state, and classical communication. CV quantum teleportation protocol, based on Braunstein-Kimble protocol [4], typically involves the following key steps:

- Entangled Resource Preparation: Alice and Bob share an entangled resource, typically a two-mode squeezed vacuum state (EPR state).
- State Interaction: Alice combines the input state ρ_{in} with her part of the entangled resource using a balanced (50:50) beam splitter.
- Bell Measurement: Alice performs homodyne measurements on the resulting modes to measure specific quadratures.
- Classical Communication: Alice sends the measurement outcomes to Bob via a classical channel.
- Displacement Operation: Bob applies displacement operations to his mode based on Alice's measurements to reconstruct the teleported state ρ_B^{out}.

B. Two-Mode Squeezed Thermal States

TMST states are fundamental resources in quantum communication and quantum teleportation protocols, particularly within CV frameworks. A TMST state is generated by applying squeezing operations to two coherent modes, which are then subjected to thermal noise inherent in realistic hardware environments. This process results in an entangled state that leverages the benefits of squeezing while contending with the challenges posed by thermal fluctuations

C. Fidelity Calculation

We consider a scenario where the TMST state, serving as the entanglement resource, is generated by Alice and then transmitted to Bob through an open-air channel. In this case, the average teleportation fidelity is calculated based on [5] as follows:

$$\begin{aligned}\bar{F}_{\text{TMST}} = \Bigg[1 + \left(\frac{1}{2} + N_c\right)\Gamma_{\text{eff}} \\ + \left(\frac{1}{2} + N_s\right)(2 - \Gamma_{\text{eff}})\cosh 2r \\ - (1 + 2N_s)\sqrt{1 - \Gamma_{\text{eff}}}\sinh 2r\Bigg]^{-1},\end{aligned} \tag{1}$$

with $\Gamma_{\text{eff}} = 1 - e^{-\alpha d}(1 - \Gamma_{\text{ant}})$ is the effective reflectivity that takes into account both the environmental factors and the antenna characteristics, N_c and N_s are the thermal photon number in channel and source, and r is the squeezing parameter. The fidelity is calculated as a function of the distance d between Alice and Bob, with the attenuation coefficient α modeling the signal degradation over distance.

III. Results and Discussion

In our numerical simulations, we have observed the fidelity of the quantum teleportation protocol using TMST states as the entanglement resource. Figure 1 illustrates our examination of four variations of the squeezing parameter, specifically $r = 0, 1, 2, 3$. These values were selected based on experimental findings in the optical frequency regime, where squeezing states have reached values around 1.7. Our findings indicate that the fidelity of the teleportation protocol is significantly influenced by the squeezing parameter. As the squeezing parameter increases, the initial fidelity at shorter distances increases. Greater squeezing intensifies the entanglement between the modes, essential for achieving high-fidelity teleportation.

Regardless of the squeezing parameter, all curves exhibit a decrease in fidelity as the distance increases. This decline is ascribed to thermal noise and attenuation in the open-air microwave channel, as detailed in Equation (1). The attenuation coefficient plays a significant role in this decline. Upon comparing the various curves, it becomes apparent that fidelity degradation is more pronounced for higher squeezing parameters. For example, the curve for $r = 3$ initially displays the highest fidelity at short distances but also undergoes the most rapid decline. This indicates that while higher squeezing enhances initial fidelity, it also renders the system more vulnerable to thermal noise over longer distances. Nevertheless, higher squeezing parameters still result in sustained higher fidelity over longer distances, such as 100 meters, compared to the curves with lower squeezing parameters.

The findings offer valuable guidance for the real-world implementation of microwave quantum communication. In short-distance scenarios, greater squeezing parameters offer higher initial fidelity. However, over longer distances, a trade-off must be struck

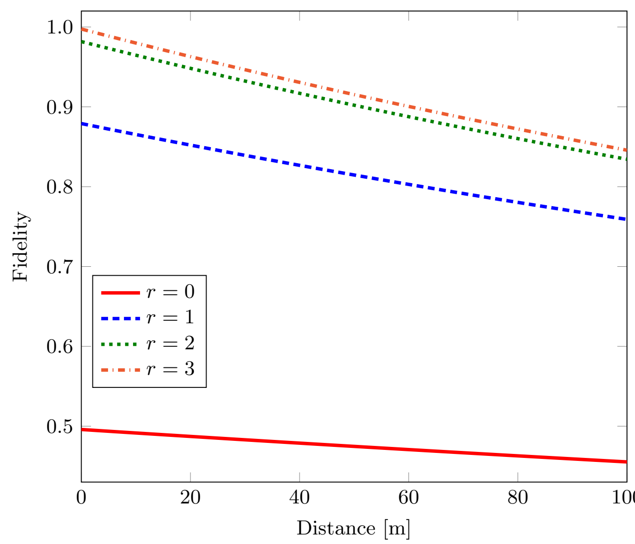

Figure 1. Fidelity of quantum teleportation using microwave TMST state with squeezing parameters $r = 0, 1, 2, 3$ over a distance.

IV. Conclusion

We evaluated the impact of squeezing parameters on quantum teleportation fidelity using microwave TMST states. Our analysis shows that higher squeezing enhances short-distance fidelity but accelerates long-range degradation. An optimal, experimentally achievable squeezing parameter is crucial for balancing initial fidelity and long-distance performance, ensuring practical implementation of efficient quantum teleportation protocols with microwave TMST states.

Acknowledgment

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korean government (MSIT) (NRF-2022R1A4A3033401), the MSIT (Ministry of Science and ICT), Korea, under the ITRC (Information Technology Research Center) support program (IITP-2024-2021-0-02046) supervised by the IITP (Institute for Information & Communications Technology Planning & Evaluation), and Kyung Hee University in 2023 (KHU-20233663).

References

[1] S. Bravyi, O. Dial, J. M. Gambetta, D. Gil, and Z. Nazario, "The future of quantum computing with superconducting qubits," *J. Appl. Phys.*, vol. 132, no. 16, Oct. 2022.

[2] M. Caleffi, M. Amoretti, D. Ferrari, J. Illiano, A. Manzalini, and A. S. Cacciapuoti, "Distributed quantum computing: A survey," *Comput. Netw.*, vol. 254, p. 110672, Dec. 2024.

[3] M. Casariego *et al.*, "Propagating quantum microwaves: towards applications in communication and sensing," *Quantum Sci. Technol.*, vol. 8, no. 2, p. 023001, Mar. 2023.

[4] S. L. Braunstein and H. J. Kimble, "Teleportation of continuous quantum variables," *Phys. Rev. Lett.*, vol. 80, no. 4, pp. 869–872, Jan. 1998.

[5] T. Gonzalez-Raya *et al.*, "Open-air microwave entanglement distribution for quantum teleportation," *Phys. Rev. Applied*, vol. 18, no. 4, p. 044002, Oct. 2022.