EPR Pairs with GHZ States Distribution

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Abstract

This paper proposes a protocol to distribute Greenberger–Horne–Zeilinger (GHZ) states utilizing shared Einstein–Podolsky–Rosen (EPR) pairs as resources in multi-party scenario. The protocol relies on the application of entangled measurement setting and local quantum operations to distribute the GHZ states among the parties. The protocol is simulated in noisy scenario and its performance is evaluated by computing the fidelity between the distributed and pure GHZ state.

I. INTRODUCTION

Greenberger–Horne–Zeilinger (GHZ) state is a central tool in quantum information technology due to its properties of entanglement and non-locality. GHZ states are multi-partite entangled states involving three or more qubits, which can be represented as a superposition of basis states as

$$|\mathrm{ghz}_M\rangle = \frac{1}{2}\left(|0\rangle^{\otimes M} + |1\rangle^{\otimes M}\right) \tag{1}$$

where M is the number of qubits. This entanglement extends across all qubits, such that a measurement on one qubit instantaneously determines the state of the others, regardless of the spatial separation. The GHZ states play important roles in quantum computing, sensing, and communication [1]–[3]. Its maximal entanglement and non-locality are exploited in protocols for quantum cryptography, distributed quantum computing, and multi-party quantum networks [1], [4]–[6].

This paper presents a protocol for distributing GHZ states by using shared Einstein–Podolsky–Rosen (EPR) pairs as resources in a multi-party setting. The protocol employs entangled measurement configurations and local quantum operations to distribute GHZ states across multiple parties. We simulate the protocol in the presence of noise and assess its performance by calculating the fidelity between the distributed state and the ideal GHZ state.

II. ENTANGLEMENT SWAPPING

Entanglement swapping allows two previously unentangled qubits to become entangled without ever interacting directly. This process is vital in scenarios where parties are spatially separated, such as in quantum communication networks, where direct interaction is not feasible [7].

Suppose that 2 parties, say, Bob and Charlie, want to share an entangled state in the form of EPR pair but they do not have direct interaction with each other. Let both Bob and Charlie have direct interaction with Alice, and each of them shares an EPR pair with Alice. Using entangled measurement setting on Alice's side and local quantum operations on both Bob's and Charlie's sides, Bob and Charlie can share an EPR pair, which is called entanglement swapping protocol.

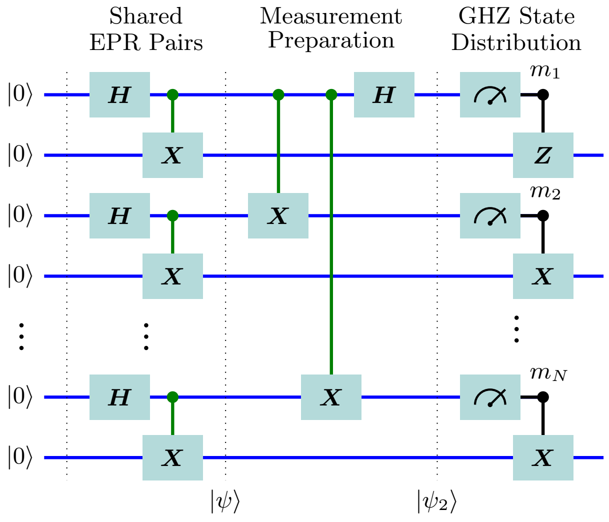

Fig. 1. Circuit Diagram.

Let the total state of Alice, Bob, and Charlie is

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right), \tag{2}$$

where Alice holds the first and third qubits, Bob holds the second qubit, and Charlie holds the fourth qubit. Alice applies the CNOT gate,

$$\boldsymbol{C}_{1,3} = |0\rangle\langle 0|_1 \otimes \boldsymbol{I}_3 + |1\rangle\langle 1|_1 \otimes \boldsymbol{X}_3, \tag{3}$$

on her qubits where the first(third) qubit is the control(target) qubit. Then, she applies Hadamard gate $\boldsymbol{H}$ on the first qubit. Alice measures her qubits on computational basis and sends the measurement outcome of the second(fourth) qubit to Bob(Charlie). Based on the received measurement outcomes, Bob(Charlie) applies $\boldsymbol{Z}(\boldsymbol{X})$ on his qubit if the measurement outcome is 1 and leaves the state unchanged otherwise. Now the protocol is completed hence Bob and Charlie share an EPR pair.

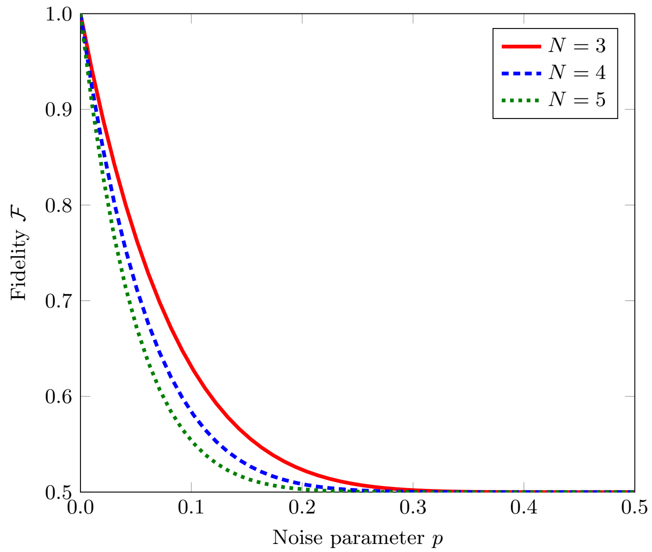

Fig. 2. Protocol performance.

III. DISTRIBUTING GHZ STATES

Let $N \geq 3$ parties where each party shares an EPR pair with Alice. Hence, the total state can be written as

$$|\psi\rangle = \bigotimes_{i=1}^{N} \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle\right). \tag{4}$$

Alice performs CNOT operations on her qubits as follows,

$$|\psi_1\rangle = \prod_{i=2}^{N} \boldsymbol{C}_{1,2i-1} |\psi\rangle . \tag{5}$$

Subsequently, Alice performs the Hadamard operation on the first qubit as

$$|\psi_2\rangle = \boldsymbol{H}_1 |\psi_1\rangle . \tag{6}$$

Finally, Alice measures her qubits using the computational basis and sends the measurement outcomes of the qubits to the corresponding parties, i.e., the measurement outcome of the ith qubit is sent to the $(i+1)/2$th party. The remaining parties perform the following operations based on the received measurement outcome,

$$|\psi_3\rangle = \boldsymbol{Z}_2^{m_1} \otimes \bigotimes_{i=2}^{N} \boldsymbol{X}_{2i}^{m_i} \mathcal{M}\left(|\psi_2\rangle\right) \tag{7}$$

where $\{m_i \in \{0,1\}\}$ are the measurement outcomes and $\mathcal{M}\left(|\psi_2\rangle\right)$ is the post-measurement state.

IV. SIMULATION RESULTS

We simulate the protocol for $N = 3, 4, 5$ where all qubits in all EPR pairs undergo phase-flip quantum noise. The dephasing quantum noise for single qubit state $|\phi\rangle$ is modeled as

$$\mathcal{E}\left(|\phi\rangle\right) = (1-p) |\phi\rangle\langle\phi| + p\boldsymbol{Z} |\phi\rangle\langle\phi| \boldsymbol{Z}. \tag{8}$$

Using a fidelity function between two quantum states $\boldsymbol{\rho}$ and $\boldsymbol{\sigma}$,

$$\mathcal{F}\left(\boldsymbol{\rho}, \boldsymbol{\sigma}\right) = \left(\operatorname{tr} \sqrt{\sqrt{\boldsymbol{\rho}}\boldsymbol{\sigma}\sqrt{\boldsymbol{\rho}}}\right)^2, \tag{9}$$

the noisy distributed GHZ state $\mathcal{E}^{\otimes N}\left(|\psi_3\rangle\right)$ is evaluated using the fidelity function with a pure GHZ state $|\mathrm{ghz}_N\rangle\langle\mathrm{ghz}_N|$.

Figure 2 shows the fidelity between the distributed and pure GHZ states in the phase-flip noise with noise parameter $p \in \{0, 0.5\}$. The fidelity for all $N = 3, 4, 5$ converges to 0.5 when the noise parameter p tends to 0.5. Furthermore, the fidelity converges faster when N increases. Note that when there is no error $(p = 0)$ the fidelity is 1 which illustrates the correctness of the protocol since the distributed GHZ state is the same as the pure one.

V. CONCLUSION

We proposed a protocol to distribute GHZ states from EPR pairs in a multi-party scenario. The protocol involves the distribution of the EPR pairs to each party, the centralized preparation of measurement setting, the communication of measurement outcomes, and the local state correction by each party. We simulate the protocol in phase-flip noise and show the correctness as well as the behavior of the protocol in a noisy scenario in terms of the fidelity between the distributed and pure GHZ states.

ACKNOWLEDGEMENTS

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korean government (MSIT) (NRF-2022R1A4A3033401), the MSIT (Ministry of Science and ICT), Korea, under the ITRC (Information Technology Research Center) support program (IITP-2024-2021-0-02046) supervised by the IITP (Institute for Information & Communications Technology Planning & Evaluation), and Kyung Hee University in 2023 (KHU-20233663).

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